A cosmological background of gravitational waves produced by supernovae in the early universe

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Abstract

Simple arguments demonstrate that the rate of supernovae within a red shift horizon $z \sim 2$ is at least of the order of 10^{10} per year or 1000 per second. This rate could be enhanced by more than an order of magnitude if the supernova rate in the early universe is enhanced as predicted by star formation models, metallicity observations, and the recent observations of an abundance of faint blue galaxies at high red shift. The gravitational waves from supernovae in the early universe create a continuous stochastic background. The amplitude of this background depends on the efficiency of gravitational wave production in supernovae, which in turn depends on the fraction of collapses which create neutron stars and black holes, the dynamics of the collapse, and the post collapse evolution of the system. It is shown that the stochastic supernova background is detectable by cross correlation of nearby detectors if the efficiency of gravitational wave production exceeds 10^{-5}. The expected spectrum is in the frequency band well suited both laser interferometer and resonant mass detectors and cross correlation between advanced bars and interferometers provides an appropriate means of detection.

1. Introduction

Supernovae, in which a star collapses to form a black hole or a neutron star, have long been considered likely sources of gravitational waves. Numerical modelling of gravitational collapses have lead to rather low estimates of the conversion efficiency to gravitational waves [1]. However, models of the post collapse evolution of neutron stars leads to higher conversion efficiency estimates. For example, Lai and Shapiro [2] consider the post-collapse evolution of a neutron star in which non-axisymmetrical instabilities lead to efficient radiation of angular momentum. They predict gravitational wave amplitudes $\sim 10^{-21}$ at ~ 30 Mpc distance ($f \sim$ few hundred hertz), corresponding to gravitational efficiency $\epsilon \sim 10^{-3}$. Houser *et al.* [3], also obtain a $\epsilon \sim 10^{-3}$ for a model which predict peak emission gravitational wave amplitudes $\sim 10^{-22}$ at ~ 20 Mpc ($f \sim$ few thousand hertz).

Here we present a first order analysis [4,5] of the combined effect of all the supernovae in the universe up to a red shift distance $z \sim 2$. We show that there can be little doubt that a continuous background of gravitational waves is created by these events. The amplitude of this background could be within the range of detectability of proposed advanced detectors if $\epsilon > 10^{-5}$. It almost certainly will mask predicted cosmological backgrounds from the big bang in the frequency range of terrestrial detectors. It provides a powerful probe of early epochs of star formation. For example, observation of the spectrum and duty cycle can allow investigation of star formation rates, supernova rates, branching ratios between black hole and neutron star formation processes, the mass distribution of black hole births, angular momentum distributions and even black hole growth mechanisms in regions of the universe completely inaccessible to individual stellar observations with electromagnetic astronomy.

2. The Supernova Duty Cycle

We first estimate the rate of supernovae in the universe as a function of distance, based on observations of supernovae in external galaxies. We assume that for distances greater than 10 Mpc the universe is sufficiently isotropic and homogeneous that the rate of supernovae beyond this distance scales proportional to the enclosed volume. Roughly 50 supernovae are discovered each year in external galaxies. Some of these discoveries are serendipitous, and they certainly do not represent a complete survey. Most of these supernovae are between 5 and 50 Mpc. As reported by Giazotto [6], the number of detected supernovae within $\sim$ 10 Mpc is $\sim$ 5 per year. The detection efficiency is probably much less than 50%, so a conservative estimate for the rate of supernovae within 10 Mpc is 10 per year. We shall use 10 Mpc as the fiducial scaling distance, denoted r_0, for our analysis. It corresponds to a mean interval T_0 between events of 0.1 years, or 3×10^6 seconds, and a mean gravitational wave amplitude h_0.

The gravitational wave burst from a gravitational collapse is expected to have duration $\tau \sim 1$ ms. The duty cycle of gravitational waves from supernovae within a distance r, denoted $D(r)$ is given by

$$D(r) = \tau / T.$$

Assuming flat space-time,

$$D(r) = \frac{\tau}{T_0} \left(\frac{r}{r_0} \right)^3 .$$

Since the mean amplitude h of a gravitational wave burst from a supernova at distance r scales as $1/r$, it follows that $r/r_0 = h_0/h$, and we may express D as a function of the mean observable gravitational wave amplitude h,

$$D(h) = \frac{\tau}{T_0} \left(\frac{h_0}{h} \right)^3 .$$

The duty cycle is a significant parameter here because it is only in the case where D approaches unity that the gravitational waves from short bursts can be considered to form a stochastic background. In addition, the ability to dig out such a background from the noise by cross correlation analysis also depends critically on the value of D. Cross correlation analysis should be able to extract the curve $D(h)$ for values of $D > 0.01$ and this, as discussed above, will allow much astrophysical information to be obtained.

To complete an estimate of $D(h)$ we need to estimate the value of h_0. It is not possible to be definitive regarding h_0 because no modelling has encompassed the full collapse and post-collapse evolution. Compare the results of Stark and Piran [1] who modelled a rotating gravitational collapse, with those of Lai and Shapiro [2] who modelled the post-collapse evolution of a rapidly rotating neutron star. It appears that the strongest gravitational radiation may occur in the post-collapse period when non-axisymmetric deformation causes essentially all the star's angular momentum to be radiated as gravitational waves. One can make several observations regarding this proposition. First, the existence of millisecond pulsars, which have very low spin down rates tells us that not all neutron stars can experience such deformations. Second, the low angular momentum of most pulsars is very conveniently explained by the Lai and Shapiro mechanism. Third, the above apparently contradictory statements can be reconciled. Very hot newly formed pulsars may be prone to strong non-axisymmetric deformations, driven by convection and rotation, while cooler stars spun up during binary interaction may remain axis-symmetric. Finally, it is interesting to note that a value of $\epsilon \sim 10^{-3}$ follows naturally if neutron stars are born with a spin frequency of 400 Hz, and lose most of their angular momentum by gravitational emission. The strain amplitude is then consistent with Lai and Shapiro's estimate of $h_0 \sim 10^{-21}$. We shall adopt these values for our analysis but finally will consider the consequence of lower values of h_0.

Figure 1 shows the form of $D(h)$. The duty cycle increases as h^{-3}. Using the above numerical values, D reaches 0.3 for $h \sim 1.5 \times 10^{-24}$. Various factors can modify this. If the supernova rate is enhanced in the early universe, as predicted from metallicity studies [7], stellar evolution models [8] and the observation of faint blue galaxies [9], D will easily exceed unity. Typical estimated enhancements of 10–100 fold mean that the mean supernova frequency will be at least $10/\tau$ for $h \sim 10^{-24}$, corresponding to a supernova rate of 10 kHz. In this regime the excess supernova rate does not increase the duty cycle (since a duty cycle above unity is meaningless). Instead it increases the rms strain amplitude leading to an *enhancement* as shown in Figure 1.

The above analysis does not consider cosmological red shift effects nor more realistic cosmological models. Since the dominant contribution to the background is from supernovae at $z \sim 2$, these effects will be substantial.

As emphasised by Schutz [10] stochastic backgrounds can be detected by cross correlation of signals from detectors spaced within less than one reduced wavelength. Fortuitously several bar-interferometer pairs exist or are planned which will be ideal for cross correlation. Cross correlation leads to an increase of the signal to noise ratio determined by the geometric mean of the detector sensitivities, and increasing as the 1/4 power of the integration time:

$$S/N = \left[S^2/(S_1 \cdot S_2) \cdot B \cdot t\right]^{1/4},$$

where S is the spectral strain amplitude of the stochastic background, S_1 and S_2 are the spectral strain sensitivities of the two detectors, B is the overlapping bandwidth of the detectors, and t is the integration time. When applied to a bar-interferometer pair Schutz showed that S/N is independent of the bar's bandwidth. For *signal = noise* the detectable stochastic background spectral density can be expressed in terms of the burst sensitivity of the instruments, h_b, for a given burst duration t:

$$h = [h_{b1} \cdot h_{b2}]^{1/2} \cdot \tau^{3/4}/t^{1/4}.$$

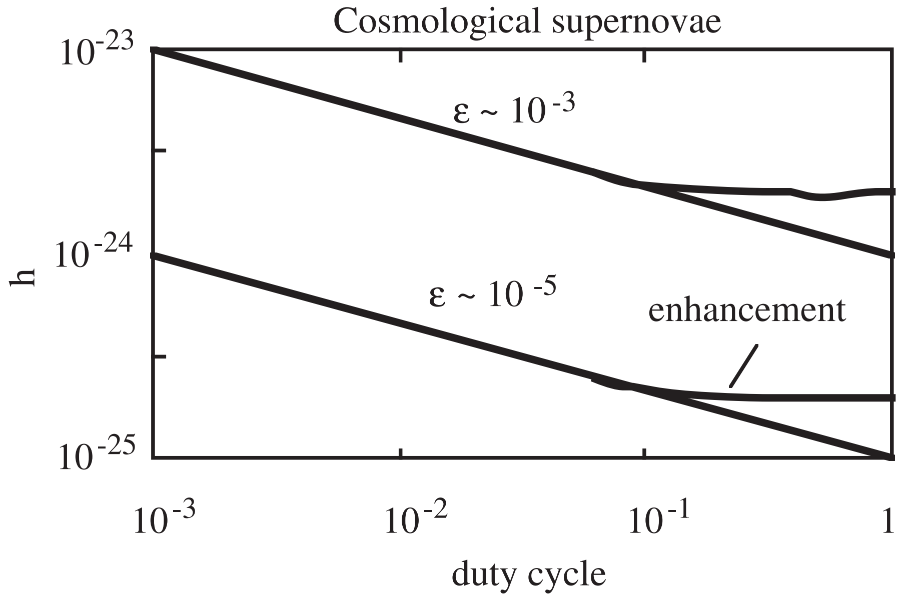

Fig. 1: The dependence of duty cycle on strain amplitude for supernovae at cosmological distances. Two curves are shown, one for gravitational wave conversion efficiency 10^{-3}, the other for two orders of magnitude lower efficiency.

Assuming $h_{b1} = h_{b2} = 10^{-21}$, and $t = 10^8$s, it follows that a background with a spectral strain amplitude of $10^{-25}/\sqrt{\mathrm{Hz}}$ can be marginally detected in 3 years observation. Advanced detectors are proposed to be able to achieve about one order of magnitude improvement over these figures.

We point out that the above analysis assumes Gaussian noise. The signals considered here are only likely to be a good approximation to Gaussian noise in the low amplitude high duty cycle regime. For higher amplitudes the background will be *popcorn* noise containing widely spaced bursts. The analysis of cross correlation for such signals is likely to be different from that given above.

3. Stochastic Background Spectrum and Energy Density

The spectrum of the stochastic background is difficult to estimate since predicted spectra are highly model dependent. We have considered two alternatives. (a) we assume a standard gravitational collapse spectrum similar to the Stark and Piran result [1], for which the energy density scales as $f^{2.5}$ up to a cut off frequency ($\sim$ 5 kHz for about 3 solar mass collapses). If the structure above cut off is ignored and $\epsilon = 10^{-3}$, this leads to an average spectrum as shown in Figure 2. (b) We assume that the dominant background is created by post collapse evolution as discussed above. In this case it is reasonable to assume only neutron star sources, and the stochastic spectrum is identical in shape to that predicted by Lai and Shapiro [2]. While the precise shape of these spectra cannot be trusted, but the average magnitude can be compared with planned detectors.

It is useful to consider the energy density of this background, by comparison with predicted cosmological backgrounds from the big bang. A background with 10^{-6} of

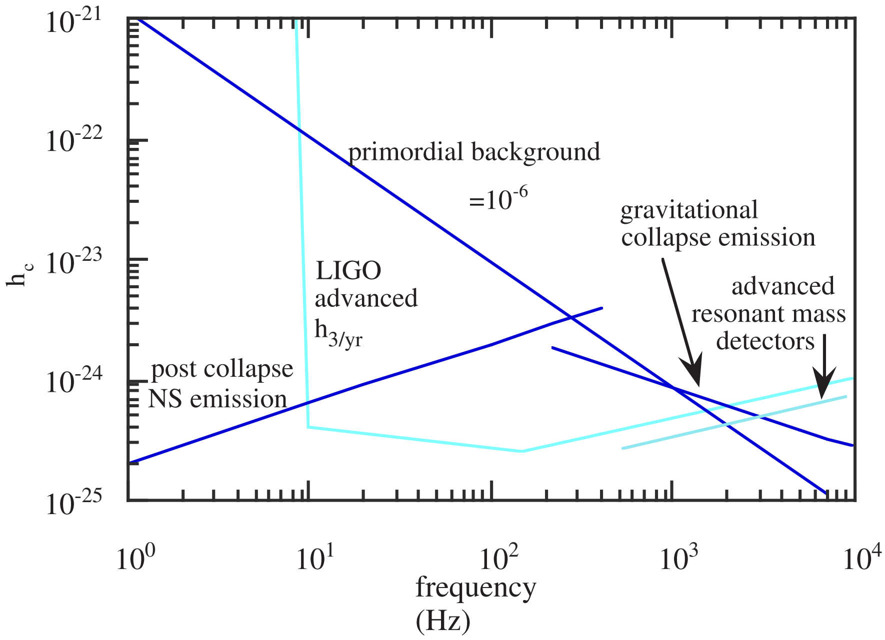

Fig. 2: Comparison of stochastic background signals [2,13] and the sensitivity of advanced laser interferometer [11,13] and resonant mass detectors [12].

closure density per decade has an amplitude of 10^{-24} at 100 Hz, falling to 10^{-25} at 1 kHz [13]. The spectrum predicted here corresponds to a comparable density. The gravitational wave energy density from supernovae Ω_{sn} can be independently estimated from the relation: $\Omega_{sn} = \Omega_m \cdot f_s \cdot f_{sn} \cdot \epsilon$, where Ω_m is the mass density of baryonic matter in the universe, f_s is the fraction of this which has taken part in star formation, f_{sn} is the mass fraction of stars which undergo supernovae and ϵ is the mean gravity wave conversion efficiency in supernovae. $\Omega_{sn} = 10^{-6}$ can be achieved if $\Omega_m \cdot f_s = 10^{-2}$, $f_{sn} = 10^{-1}$ and $\epsilon = 10^{-3}$.

4. Conclusion

The cosmological background of gravitational waves from supernovae is certain to exist. Its magnitude could be in the range of 10^{-24} to 10^{-25}, detectable by advanced instruments. This implies that a relatively large fraction of the matter in the universe must pass through supernovae, and could imply that more baryonic matter than conventionally believed is locked up in collapsed objects. This topic is worthy of further consideration. The analysis presented here is a first approximation. However, it leads to one strong and interesting conclusion. Pairs of detectors capable of detecting identical burst events at 30 Mpc are capable of detecting the stochastic background of such events at cosmological distances as long as the duty cycle for this background approaches unity. Further study should examine the cross correlation of non-Gaussian noise. Red shift effects, and better collapse models should be used to model the spectrum, and signal processing methods for extracting spectral and duty cycle information need to be developed. Future detectors, whether resonant mass or laser interferometers, should be located within about 50 km of another detector to allow cross correlation to take place.

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