Post-Newtonian smooth particle hydrodynamics

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Abstract

The smoothed particle hydrodynamics (SPH) method has been extensively applied to predicting the behaviour of compressible Newtonian fluids for astrophysical problems. In this paper it is extended to the post-Newtonian (PN) approximation to the fully general relativistic equations retaining perturbations up to $1/c^2$. An obvious advantage of this approach is that standard numerical methods for Newtonian hydrodynamics can be extended easily to the post-Newtonian equations unlike the situation for the equations governing general relativity. The post-Newtonian SPH method has been validated against a number of test cases including relativistic polytropes and general relativistic hydrodynamical collapse calculations. Preliminary results have been obtained for the collapse of rotating stellar cores. In general, the method is applicable to general relativistic astrophysical problems such as stellar core collapse and the coalescence of neutron stars in binaries. Both of these problems have previously been treated mainly by Newtonian methods.

1. Introduction

With the promise of more sensitive gravitational wave detectors such as the laser interferometry based systems like LIGO and VIRGO, and improvements in the niobium-bar technology, such as sapphire sensors, it now seems likely that finally it will be possible to detect gravitational radiation from some energetic astrophysical events. The two most likely candidates appear to be neutron stars in binaries undergoing coalescence and supernova, although in the latter case the estimates of the energy carried off by gravitational radiation differ widely between different simulations (Blair & Ju 1996). It is estimated that the new generation of laser interferometry based detectors, such as LIGO, (e.g., Cutler *et al.,* 1992) could detect about 3 neutron star mergers per year (Narayan, Piran & Shemi 1991, Phinney 1991), and an advanced LIGO detector could observe perhaps 70 events per year (Finn & Chernoff 1993).

Since these events are certainly strongly influenced by general relativistic effects it is not clear that a Newtonian treatment will suffice. However, there have been problems with fully general relativistic methods; especially with the development of singularities. Although a post-Newtonian treatment may not capture the full effects of general relativity it has some advantages. In particular, the methods developed for solving the equations of Newtonian dynamics can be easily extended to the post-Newtonian equations, and a comparison of Newtonian and post-Newtonian simulations will indicate the nature of the general relativistic corrections.

Gravitational radiation first appears in the sequence of post-Newtonian expansions at order $2\frac{1}{2}$, i.e., $1/c^5$. Nevertheless, it is important to include this feature for astrophysical problems since gravitational radiation emissions provide new information about the nature of extreme astrophysical events. Blanchet *et al.,* (1990) take the view that it should be included because it represents qualitatively new physics not included at PN order, and it has a cumulative effect which can have an important overall influence on the dynamical evolution of matter. For coalescing neutron stars in a binary, observations of the waveform over the final 1000 orbits can provide accurate indirect measurements of the masses and spins as well as the orbital parameters (Lai, Rasio & Shapiro 1993).

For neutron star mergers, considerable effort has been expended on developing computational tools to predict the gravitational wave signal during the final few thousand orbits as the frequency changes between about 10 and 1000 s^{-1}. This can be achieved with point mass models for the neutron stars and a high-order PN expansion; including terms up to $O(1/c^6)$ or higher (Will 1996). During the final stages of the inspiral, when the size of the orbit becomes comparable with the stellar radii, hydrodynamical effects become significant and the gravitational waveforms provide information about the internal structure of the neutron stars (as well as general relativistic effects). It appears that narrow-band, special-purpose detectors can determine the waveform during this period (Meers 1988, Strain & Meers 1991). Even a determination of the maximum frequency reached will provide information on the radii of the neutron stars and therefore place constraints on the nuclear equation of state (Cutler *et al.,* 1993).

Various groups have performed neutron star coalescence calculations using a variety of computational techniques. Lai, Rasio & Shapiro (1993, 1994abc), constructed and used compressible generalisations of the *Darwin-Riemann* classical ellipsoidal distributions to study the final stages of the evolution. They also crudely included some first-order general relativistic effects. A number calculations have been performed with Newtonian finite-difference codes and have concentrated on gravitational radiation emission. These have employed a polytropic equation of state and gravitational radiation is included by a backreaction formula (Oohara & Nakamura 1989, 1990, Nakamura & Oohara 1989, 1990 and Shibata, Nakamura & Oohara 1992). Calculations have been performed with non-relativistic SPH codes (Rasio & Shapiro 1992, Davies *et al.,* 1994, Zhuge, Centrella & McMillan 1994), the latter including a calculation of the gravitational radiation signature using the quadrupole formula. Ruffert, Janka & Schaefer (1995) have also performed Newtonian simulations with a code based on the piecewise parabolic method. Finally, Wilson, Matthews & Marronetti (1996) performed preliminary simulations based on a method which includes *most* of the effects of general relativity. For these simulations, the neutron stars collapse to black holes prior to the final merger.

Apart from neutron star coalescence, another likely candidate for significant gravitational wave emission, is stellar core collapse, either from supernova explosions or accretion-induced collapse of white dwarfs. The scenario here seems much more uncertain as it depends on whether axisymmetry is maintained during the evolution. Thompson (1984) and more recently Houser, Centrella & Smith (1994), have used an SPH code to show that for certain differentially rotating stellar cores with a polytropic equation of state, the core undergoes a bar instability. The latter paper shows that the gravitational radiation amplitude is large enough to be detectable with the new generation of gravitational wave detectors for sources within the galaxy.

The aim of this paper is to demonstrate how the post-Newtonian equations can be treated in the context of the SPH method. Actually, a post-Newtonian version of the SPH method was first developed some time ago (Thompson 1984) and applied to the simulation of the collapse of rotating stellar cores. At that time the computations suffered from lack of resolution due to limited computational speed—typically restricting the particle number to only 500. Even so, for differentially rotating initial models, a bar instability developed in agreement with subsequent simulations (e.g., Houser, Centrella & Smith 1994, Lai & Shapiro 1994). Since then computer speed has increased by three orders of magnitude which has made it possible to achieve acceptable resolution for such simulations even on workstations.

2. The Post-Newtonian Equations

The particular form described in this paper are due to Chandrasekhar (1965) and are appropriate for an ideal fluid. These approximate equations are expansions in the parameter $1/c^2$ (where c is the speed of light). The zeroth order equations give Newtonian hydrodynamics while retaining the first-order terms gives the PN approximation. The equations governing conservation of baryon number, internal energy and momentum, can be expressed as

$$\frac{\partial \rho^*}{\partial t} + \frac{\partial}{\partial x_\alpha}(\rho^* v_\alpha) = 0,$$

$$d\Pi - \frac{P}{\rho^2} d\rho = 0$$

and

$$\frac{\partial}{\partial t}(\sigma v_\alpha) + \frac{\partial}{\partial x_\beta}(\sigma v_\beta v_\alpha) = -\frac{\partial}{\partial x_\alpha}[(1+\frac{2U}{c^2})P] + \rho\frac{\partial U}{\partial x_\alpha} - \frac{4}{c^2}\rho\frac{d}{dt}(v_\alpha U - U_\alpha)$$

$$-\frac{4}{c^2}\rho v_\beta \frac{\partial U_\beta}{\partial x_\alpha} - \frac{1}{2c^2}\frac{\partial^3 \Psi}{\partial x_\alpha \partial t^2} + \frac{2}{c^2}\rho(\phi\frac{\partial U}{\partial x_\alpha} + \frac{\partial \Phi}{\partial x_\alpha}).$$

where

$$\rho^* = \rho(1 + \frac{1}{c^2}(\frac{v^2}{2} + 3U)),$$

$$\sigma = \rho(1 + \frac{1}{c^2}(v^2 + 2U + \Pi + \frac{P}{\rho})),$$

$$\phi = v^2 + U + \frac{1}{2}\Pi + \frac{3}{2}\frac{P}{\rho},$$

the PN potentials are defined by

$$\nabla^2 U = -4\pi G\rho,$$

$$\nabla^2 U_\alpha = -4\pi G\rho v_\alpha,$$

$$\nabla^2 \Phi = -4\pi G\rho\phi$$

and

$$\Psi = -G\int \rho(\mathbf{x}')|\mathbf{x}-\mathbf{x}'|d\mathbf{x}'.$$

The remaining variables represent

ρ	rest mass density,
ρ^*	baryon mass density,
v_α	α component of three velocity,
Π	specific thermal energy,
P	pressure,
U	gravitational potential,
G	universal gravitational constant,
c	speed of light.

To express the LHS of the momentum equation as a total time derivative and to remove the second-order partial time derivative term from the RHS, the following relationships are useful:

$$\rho^*\frac{d}{dt}(\frac{\sigma}{\rho^*}v_\alpha) = \frac{\partial}{\partial t}(\sigma v_\alpha) + \frac{\partial}{\partial x_\beta}(\sigma v_\alpha v_\beta)$$

and

$$\frac{\partial^3\Psi}{\partial^2 t\partial x_\alpha} = \frac{d}{dt}(U_\alpha - \Psi_\alpha) - v_\mu\frac{\partial}{\partial x_\mu}(U_\alpha - \Psi_\alpha),$$

where

$$\Psi_\alpha = G\int \rho(\mathbf{x}')v_\beta(\mathbf{x}')\frac{(x_\alpha - x'_\alpha)(x_\beta - x'_\beta)}{|\mathbf{x}-\mathbf{x}'|^3}d\mathbf{x}'.$$

These allow the momentum equation can be manipulated into the form

$$\frac{d}{dt}(\zeta v_\alpha) = -\frac{P}{\rho^{*2}}(1+\frac{2U}{c^2})\frac{\partial\rho^*}{\partial x_\alpha} - \frac{\partial}{\partial x_\alpha}((1+\frac{2U}{c^2})\frac{P}{\rho^*}) + \frac{\rho}{\rho^*}\frac{\partial U}{\partial x_\alpha} + \frac{4}{c^2}\frac{dU_\alpha}{dt}$$

$$-\frac{4}{c^2}v_\beta\frac{\partial U_\beta}{\partial x_\alpha} + \frac{2}{c^2}(\phi\frac{\partial U}{\partial x_\alpha} + \frac{\partial\phi}{\partial x_\alpha}) + \frac{1}{2}\frac{d}{dt}(U_\alpha - \Psi_\alpha) - +\frac{1}{2}v_\mu\frac{\partial}{\partial x_\mu}(U_\alpha - \Psi_\alpha),$$

where the following new variable has been introduced

$$\zeta = (\frac{\sigma}{\rho^*} + \frac{4}{c^2}U) = 1 + \frac{1}{c^2}(\frac{1}{2}v^2 + \Pi + \frac{P}{\rho} + 3U).$$

In deriving these equations, any perturbations of order higher than $1/c^2$ have been consistently ignored. In addition, the pressure force term has been expressed in a form resulting in discrete momentum, angular momentum and energy (for h constant) conservation in the SPH approximation.

The 'mass' conservation equation is based on the variable ρ^* rather than ρ. The former is just the baryon number density times the mass per baryon. Thus the kernel estimation procedure underpinning the SPH method must be based on ρ^* rather than ρ.

The post-Newtonian mass is defined as

$$M^* = \int \rho^*(\mathbf{x}) d\mathbf{x}$$

and is conserved. Consistent with the Newtonian SPH method, the (PN) density (ρ^*) can be approximated by

$$\rho^*(\mathbf{x}) = \frac{M^*}{N} \sum_{j=1}^{N} W(|\mathbf{x} - \mathbf{x}'_j|, h).$$

Estimates for other variables can be made in the usual way. For example, the density can be approximated by

$$\rho = \frac{M^*}{N} \sum_{j=1}^{N} W(|\mathbf{x} - \mathbf{x}'_j|, h) \frac{\rho_j}{\rho^*_j}.$$

The PN *potentials*, U, U_α and Φ, (and the forces) can be evaluated by replacing the source terms in the Poisson equations by their SPH estimates and integrating exactly (once the functional form of the kernel is chosen explicitly). Similarly, the PN potentials Ψ_α can be treated in a similar fashion, but starting from the integral expression rather than a Poisson equation. A complication is that the source terms depend on ρ rather than ρ^*. This means that the source term for the potentials involve the gravitational potential U and the velocity field. In practice, these contributions should be small if the PN approximation is valid. They can be treated numerically by iterating or predicting the contribution from a previous timestep using an Adams-Bashforth type predictor. This latter approach has been adopted for the current implementation. This complication is part of a more general problem associated with the PN approximation and GR in general. That is, the problem of setting up initial conditions for a simulation. With the current form of the momentum equations, initially the time derivatives of ζ, U_α and $(U_\alpha - \Psi_\alpha)$ are unknown. In addition, as mentioned previously, the source term for the Poisson equation for U depends on U and v^2. For core collapse simulations these are not serious problems because at white dwarf densities the PN corrections are negligible. On the other hand, for neutron star coalescence simulations, the neutron stars themselves are moderately relativistic and consequently these PN corrections cannot be ignored. In that case, the initial conditions need to be computed first by iteration.

For the validation studies described in this paper, a variable timestep leap-frog scheme was used for the time integration. An artificial viscosity term was added to the pressure force (Lattanzio *et al*, 1984) to limit particle interpenetration and to handle shocks.

For core collapse simulations the length scale changes by two orders of magnitude. To maintain resolution the smoothing length was varied with time, typically it was chosen to depend on the reciprocal of the gravitational potential energy. In general, the smoothing length can also be a function of space.

3. A Static Test: Computation of Equilibrium Polytropes

This test case has been used extensively in the past to validate Newtonian SPH codes. For the GR case, Tooper (1964) derived the exact solutions for spherically symmetric equilibrium configurations using a polytropic equation of state. That work relied on the equation of hydrostatic equilibrium for a spherically symmetric star first derived by Oppenheimer and Volkoff (1939) in the form

$$\frac{dP}{dr} = -\frac{(\epsilon + P)(m + 4\pi r^3 \epsilon)}{r(r - 2m)}.$$

Here, the symbols have the same meanings as for the PN equations described above, ϵ is the 00 component of the energy-momentum tensor, which includes the rest-mass density and internal energy contributions, and m is the interior mass. The polytropic equation is of the form

$$P = \kappa \epsilon^{1+\frac{1}{n}},$$

where n is the polytropic index. The exact solutions can be computed simply by integrating out from the centre of the polytrope assuming scaled density of unity and zero density gradient. GR polytropes are governed by a parameter $\sigma^* = P_c/(\epsilon_c c^2)$, with the subscript c referring to central values. This is a measure of the influence of general relativistic corrections and is closely related to the maximum specific thermal and gravitational energies.

SPH polytropes are constructed by the dividing total PN mass equally between the SPH particles, adding a damping term and letting the particles settle under the influence of the forces acting. As with Newtonian SPH polytropes the central density is not necessarily unity. However if the central density is rescaled to unity and the radial coordinate adjusted to preserve the total mass, a direct comparison can be made.

Figure 1 shows such a comparison for PN polytropes for various values of σ^*. (The Newtonian case corresponds to $\sigma^* = 0$.) For $\sigma^* = 0.0$ and 0.05 the density profiles calculated directly from the equation of hydrostatic equilibrium and the SPH results agree to within graphical error. At $\sigma^* = 0.10$, there is a slight difference between the curves. For that case the maximum gravitational potential is 0.22 c^2 so the PN equations may not be an adequate description of GR anyway. The Newtonian result is shown on the same axes for comparison to demonstrate the substantial influence of GR for this σ^*.

4. A Dynamical Test: Comparison with Van Riper Collapse Calculations

Van Riper (1978,1979) investigated the effects of general relativity on bodies close to (Newtonian) neutral stability. He examined the collapse of $n = 3$ polytropes using both

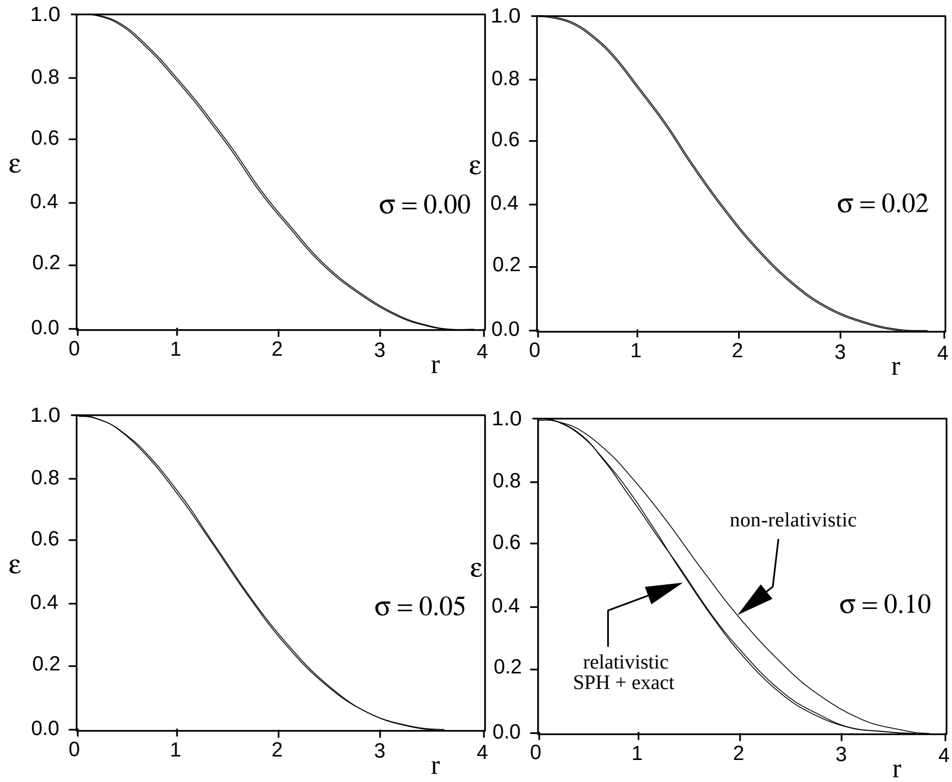

Figure 1: Comparison of PN SPH and exact density profiles for different values of the PN parameter σ. For $\sigma = 0$, 0.02 and 0.05 the exact and SPH calculations agree to graphical accuracy. For the most relativistic case, $\sigma = 0.10$ there is a slight difference. In that case the non-relativistic profile is shown for comparison.

a Newtonian and GR spherically symmetric hydrodynamical code. Such models have obvious relevance to the collapse of stellar cores as they reach the Chandrasekhar limiting mass.

The procedure used was as follows. A 1.4 solar mass polytrope star with a central density of 4.0×10^9 g/cm^3, (modelling a precollapse degenerate stellar core at the stability limit), was suddenly forced to collapse because of a reduction in the pressure force by a factor of d, only slightly less than unity. A typical parameter characterizing the behaviour as a function of d was the time taken to reach a central density of 10^{12} g/cm^3. Newtonian and GR predictions were compared.

As a validation of the PN SPH code some of these runs were repeated. Three different pressure reduction factors were used. Table 1 shows a comparison of the results obtained by Van Riper, and both a Newtonian and PN SPH code. The particle number was limited to 400 for these runs. Even so, the SPH predictions are typically within about 10% for both the Newtonian and GR cases.

This is a sensitive test for SPH because the pressure and gravitational forces are only out of balance by a factor $(1-d)$ which ranged between 0.15 and 0.54%. The extra 'forces' due to the general relativistic corrections therefore play a significant role; especially in the later stages of the collapse. Also, it should be borne in mind that the SPH code is fully three-dimensional, while the Van Riper models are one-dimensional.

Pressure Reduction factor d	model type	collapse time (msec) Van Riper	SPH
0.9985	Newtonian	1478	1360
	GR	643	740
0.9965	Newtonian	1046	950
	GR	589	640
0.9946	Newtonian	854	770
	GR	547	570

Table 1

5. Other tests

Other validation studies have been performed such as general relativistic periastron advance and duplication of the collapse simulations of May and White (1966, 1967), but these will not be described here. For those and the comparisons presented in this paper, only a small number of particles was used (typically 500-1000). Despite this, the SPH predictions were typically accurate to within about 10% of the literature or accepted values, consistent with the error expected from using a limited number of particles.

6. Recent Improvements

Since the time when this version of PN SPH was developed (readily accessible) computers have increased in computational speed by perhaps a factor of 1000. At the time of development, standard techniques like tree and multipole methods for computing the force terms were less efficient than direct summation due to the small number of particles. However, with present desktop workstations, preliminary results indicate that using tree-codes for the gravitational force terms and potentials on standard workstations, it should be possible to use approximately 50000 SPH particles for simulations of core collapse or neutron star mergers.

Tree codes work by grouping neighbouring distant particles together and computing the gravitational effect in terms of the first few terms of a multipole expansion. Close particles are still treated by direct summation. In fact, this is done in a hierarchical manner using a *tree* in which groups of neighbouring particles are subdivided into smaller groups (which are further subdivided, etc) and for each group the first terms of the multipole expansion are calculated. In evaluating the contribution to the gravitational force for each SPH particle, the tree is ascended and for each group of contributing particles the error induced by using the truncated multipole expansion instead of direct summation is computed. If the error is acceptable then the multipole expansion is used, else the tree is ascended (to the next highest level) and the comparison is repeated. If the highest level is reached and the multipole error test fails, direct summation is used for that group of particles.

An advantage of tree codes over grid-based fast Poisson solvers, such as multigrid, is in their ability to control the error more precisely. For grid-based solvers, the mass of each particle is effectively smeared over a grid cell and the grid resolution will clearly influence the 'smoothing of the particles: it is difficult to quantify the effect on the accuracy. With a tree code, the (SPH) smoothing of the particle potential can be included directly, so the error is due only to the neglect of higher-order multipole moments from groups of particles at a distance. The maximum acceptable error can be set explicitly. A further advantage is that tree codes can solve for variables which do not satisfy Poisson equations just as easily, such as the PN potential Ψ.

Tests indicate that for a variety of particle distributions, the (Newtonian) gravitational forces can be calculated an order of magnitude faster with a tree-code than by direct summation. In this case the maximum pointwise error was at most only a few tenths of a percent. Since the PN force corrections are required to be small relative to Newtonian forces, it is acceptable to allow a larger fractional error for those terms which, in turn, significantly reduces the relative computer time requirements.

7. Conclusions

A version of the SPH method based on the post-Newtonian expansion has been developed and validated against both static and dynamic general relativistic test cases. Even with limited particle number the predictions are generally accurate to within about ten percent. Fast Poisson solvers, such as tree or multipole codes can be used to calculate both the Newtonian and post-Newtonian force contributions. This will allow good resolution to be achieved for problems such as neutron star coalescence and stellar core collapse.

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