# Solving the fine-tuning problem of inflation

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**Abstract**

I describe a recently derived stochastic approach to inflaton dynamics which provides a framework for understanding the dynamics of a quantum scalar field driving an inflationary phase. This theory can address some serious problems associated with the conventional textbook approach. Specifically, it can address the quantum-to-classical transition problem, and it will be shown to lead to a dramatic easing of the fine tuning constraints that have plagued inflation theories.

The inflationary universe scenario asserts that, at some very early time, the universe went through a de Sitter phase expansion with scale factor $a(t)$ growing as $e^{Ht}$. Inflation is needed because it solves the horizon, flatness and monopole problems of the very early universe and also provides a mechanism for the creation of primordial density fluctuations. For these reasons it is an integral part of the standard cosmological model [1].

The inflationary phase is driven by a quantum scalar field with a potential $V(\Phi)$, that can take on many different forms that satisfy the 'slow roll' conditions. In the conventional approach to inflaton dynamics [1], the inflaton field $\Phi$ is first split into a spatially homogeneous piece and an inhomogeneous piece

$$\Phi(\mathbf{x}, s) = \phi(s) + \psi(\mathbf{x}, s). \tag{1}$$

The dynamics of the $\phi$ is then postulated to obey the classical 'slow roll' equation of motion

$$\dot{\phi} + \frac{V'(\phi)}{3H} = 0. \tag{2}$$

This equation governs the dynamics of $\phi$ which drives the inflationary phase. It is also possible to discuss the generation of primordial density fluctuations using $\psi$. Assuming that $\phi \gg \psi$, it can be shown that $\psi$ is described by a free massless minimally coupled quantum scalar field. During exponential inflation the quantum fluctuations of $\psi$ grow as [2]

$$\langle \psi^2 \rangle = (2\pi)^{-2} H^3 t. \tag{3}$$

These quantum fluctuations are then identified with the classical fluctuations which generate primordial density fluctuations [1, 3]. It is important to note here that interactions between the coarse-grained field $\phi$ and its fluctuations $\psi$ are ignored. This is possible in this approach because the density fluctuations are directly identified with $\langle \psi^2 \rangle$.

Consistent with the conventional approach above is the 'Stochastic Inflation' program initiated by Starobinsky [4] and further developed by others [5]. In this case the field $\phi$ obeys

$$\dot{\phi} + \frac{V'(\phi)}{3H} = \frac{H^{3/2}}{2\pi} F_w(t), \tag{4}$$

where $F_w(t)$ is a zero mean gaussian white noise source of unit amplitude. In this case $\phi$ describes the field $\Phi$ coarse-grained over a volume determined by the apparent de Sitter horizon. We will refer to $\phi$ as the local order parameter. In this method the observable universe is comprised of many patches each with its own local order parameter whose dynamics obeys (4). Spatial inhomogeneities arise because the local order parameter in each patch can take on different values by virtue of the noise in (4). Equation (4) has been the basis for many applications including studies of the generation of primordial density fluctuations [6, 7] and the very large scale structure of the universe [8].

A problem with the conventional approach is that it is assumed, without justification, that the local order parameter $\phi$ can be treated as a classical order parameter, and that the quantum fluctuations of $\psi$ are equivalent to classical fluctuations. Since $\phi$ and $\psi$ are treated as de-coupled closed quantum systems it is impossible for this method to explain the quantum-to-classical transition of $\phi$ and $\psi$ from first principles. Another more serious problem comes from directly identifying the quantum fluctuations $\langle\psi^2\rangle$, with the classical fluctuations that generate primordial density fluctuations. This scheme leads to an overproduction of primordial density fluctuations which can only be avoided by unnaturally fine-tuning the coupling constants in the inflaton potential. This is the well known fine-tuning problem of inflation.

Several authors have previously suggested that these problems arise because the conventional approach to calculating primordial density fluctuations is inconsistent with the established methods of non-equilibrium statistical physics. This was first pointed out by Hu and Zhang [9] and further developed in [10, 11, 12] (see also Lombardo and Mazzitelli [13] and Morikawa [14]). Morikawa [15] first suggested that this inconsistency was the origin of the fine-tuning problem. This was explored in more detail by Calzetta and Hu [16] and most recently by Calzetta and Gonorazky [17].

While the conventional approach may be the only possible one for a free field, in [18] an alternative has been developed for interacting fields which does address the problems outlined above. The theory is similar in style to the conventional stochastic inflation program but differs in a fundamental way. In this theory we no longer identify the quantum fluctuations $\langle\psi^2\rangle$ directly with the classical fluctuations that generate primordial density fluctuations. The new role of the field $\psi$ is to provide a noise source (via backreaction) in the quantum dynamics of the local order parameter $\phi$. This is nothing but an application of the well known quantum Brownian motion paradigm of non-equilibrium statistical physics. The field $\psi$ plays the role of an environment which couples to the system $\phi$ and indirectly generates fluctuations $\delta\phi$ in the system. This environmental noise will generate quantum decoherence which is the process that leads to entropy generation and the quantum-to-classical transition of the order parameter and its fluctuations. We then identify the resulting classical fluctuations of the local order parameter $\delta\phi$, as those which lead to density fluctuations, rather than the quantum fluctuations derived directly from $\langle\psi^2\rangle$. Clearly in this approach the interaction between $\phi$ and $\psi$ is critical. As well as addressing the quantum-to-classical transition problem, this theory leads to a dramatic easing of the fine tuning constraints, a problem that has plagued the conventional approach to inflaton dynamics.

The main result is that, for a minimally coupled scalar field in a de Sitter phase, the quantum dynamics of the local order parameter $\phi$ can be described by the relatively simple stochastic quantum mechanical Hamiltonian [18]

$$H(t) = \frac{p^2}{2e^{3Ht}} + e^{3Ht}V(\phi) - \frac{H^2}{8\pi^3}e^{3Ht}\, V''(\phi)F_c(t), \tag{5}$$

where $p = e^{3Ht}\dot{\phi}$ is the canonical momentum and $p$ and $\phi$ obey the usual quantum mechanical commutator. $F_c(t)$ is a zero mean gaussian coloured noise of unit amplitude with a correlation time of the order $H^{-1}$. This result is valid for a general inflaton potential. The origin of the noise is the backreaction of quantum fluctuations with wavelengths shorter than the coarse-graining scale. The noise is of a multiplicative nature because its origin is the mode-mode coupling induced by the self-interaction of the inflaton. For a free field the stochastic term vanishes. This is because the environment $\psi$ and the system $\phi$ now decouple and the conventional situation is recovered. Major simplification was made by ignoring information about spatial correlations between the order parameters of different regions. This allows a description based on a single degree of freedom. Further significant simplification was obtained by invoking the standard slow roll assumptions. This makes it possible to show that the potential renormalisation and non-local dissipation terms are negligible.

In the semiclassical limit the dynamics of the local order paramter is described by [18]

$$\dot{\phi} + \frac{V'(\phi)}{3H} = \frac{H^{1/2}}{\sqrt{864}\pi^3}V'''(\phi)F_w(t). \tag{6}$$

$F_w$ is a white noise of unit amplitude that is interpreted in the Stratonovich sense (since it is an approximation to a coloured noise). We also interpret the noise in (4) the same way, though in this case one is also free to use the Ito interpretation. In deriving this equation from (5) we have invoked the slow roll conditions to neglect the inertial term and approximate the intrinsically coloured noise by a white noise.

In [19] we have studied an exact solution to (6) for the quartic potential

$$V(\phi) = \lambda\phi^4/4 \tag{7}$$

of chaotic inflation, and used it to study the generation of primordial density fluctuations. We found that on both sub and super-horizon scales the theory predicts gaussian fluctuations to a very high accuracy along with a near scale-invariant spectrum. This is in general agreement with observations. Of most interest is that the amplitude constraint is found to be satisfied for $\lambda \sim 10^{-5}$ rather than $\lambda \sim 10^{-14}$ of the conventional theory. This represents a dramatic easing of the fine-tuning constraints, a feature likely to generalise to a wide range of potentials. These results were based on equation (6), the classical limit of the theory. The other great advantage of this theory is that it leads naturally to a description of the inflaton whose dynamics is described by the quantum open system (5). This allows the quantum-to-classical transition to occur as a non-equilibrium quantum statistical process (decoherence), rather than being simply postulated as in the conventional approach. Work on this issue is currently in progress.

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## References

[1] T. Padmanabhan, *Structure Formation in the Early Universe*, Cambridge University Press (1993); P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton University Press, N.J. (1993); A.R. Liddle and D.H. Lyth, Phys. Rep. **231**, 1 (1993); A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic Publishers GmbH, Chur, Switzerland, (1990); E.W. Kolb and M.S. Turner, *The Early Universe*, Addison Wesley (1990).

[2] A. Vilenkin and L. Ford, Phys. Rev. **D26**, 1231 (1982); A. Linde, Phys. Lett. **116B**, 335 (1982).

[3] A. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982); S. W. Hawking, Phys. Lett. **115B**, 295 (1982); J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. **D28**, 629 (1983); R. Brandenberger, R. Kahn and W. Press, Phys. Rev. **D 28**, 1809 (1983); V. Mukhanov, H. Feldman and R. Brandenberger, Phys. Rep. **215**, 203 (1992); R. Brandenberger, Nucl. Phys. B. **245**, 328 (1984); A. H. Guth and S. Y. Pi, Phys. Rev. **D32**, 1899 (1985).

[4] A. A. Starobinsky, in *Field Theory, Quantum Gravity and Strings*, ed. H. J. de Vega and N. Sanchez (Springer, Berlin 1986).

[5] S. J. Rey, Nucl. Phys. **B284**, 706 (1987); J. M. Bardeen and G. J. Bublik, Class. Quan. Grav. **4**, 473 (1987); M. Morikawa, Phys. Rev. **D42**, 1027 (1990); H. E. Kandrup, Phys. Rev. **39**, 2245 (1989); A. Hosoya, M. Morikawa and K. Nakayama, Int. J. Mod. Phys. A **4**, 2613 (1989); Y. Nambu, Phys. Lett. B **276**, 11 (1992); M. Mijic, Phys. Rev. **D49**, 6434 (1994); F.R. Graziani, Phys. Rev. **D38**, 1122 (1988);**D38**, 1131 (1988);**D38**, 1802 (1988); **D39**, 3630 (1989); O.E. Buryak, Phys. Rev. D **53**, 1763 (1996).

[6] I. Yi and E. T. Vishniac, Phys. Rev. **D43**, 5295 (1993); Phys. Rev. **D45**, 3441 (1992); D.S. Salopek and J.R. Bond, Phys. Rev. **D43**, 1005 (1991); S. Mollerach, S. Matarrese, A. Ortolan, and F. Lucchin, Phys. Rev. **D44**, 1670 (1991); G.V. Chibisov and Yu. V. Shtanov, Int. J. Mod. Phys. **A13**, 2625 (1990); D.S. Salopek, J.R. Bond

and J.M. Bardeen, Phys. Rev. **D40**, 1753 (1989); S. Matarrese, A. Ortolan and F. Lucchin, Phys. Rev. **D40**, 290 (1989); A. Ortolan, F. Lucchin and S. Matarrese, Phys. Rev. **D38**, 462 (1988).

[7] I. Yi, E.T. Vishniac and S. Mineshige, Phys. Rev. **D43**, 362 (1991).

[8] A. Linde, D. Linde and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994); Phys. Lett. **B307**, 25 (1993); Phys. Lett. **B345**, 203 (1995); Y. Nambu and M. Sasaki, Phys. Lett. B **219**, 240 (1989); Y. Nambu, Prog. Theor. Phys **81**, 1037 (1989).

[9] B. L. Hu and Y. Zhang, "Coarse-Graining, Scaling, and Inflation" Univ. Maryland Preprint 90-186 (1990); B. L. Hu, in *Relativity and Gravitation: Classical and Quantum* Proceedings of SILARG VII, Cocoyoc, Mexico, Dec. 1990. eds. J. C. D' Olivo *et al* (World Scientific, Singapore, 1991).

[10] B. L. Hu, J. P. Paz and Y. Zhang, in *The Origin of Structure in the Universe*, ed. by E. Gunzig and P. Nardone (Kluwer, Dordrecht 1993).

[11] B. L. Hu, in *Quantum Physics and the Universe*, ed. Namiki, K. Maeda, et al (Pergamon Press, Tokyo, 1993). Vistas in Astronomy **37**, 391 (1993).

[12] B. L. Hu, in *Proc. Second International Workshop on Thermal Fields and Their Applications*, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991).

[13] F. Lombardo and F.D. Mazzitelli, Phys. Rev. D **53**, 2001 (1996).

[14] M. Morikawa, Prog. Theor. Phys **93**, 685 (1995).

[15] M.Morikawa, in *Quantum Physics and the Universe*, ed. Namiki, K. Maeda, et al (Pergamon Press, Tokyo, 1993). Vistas in Astronomy, **37**, 87 (1993).

[16] E. Calzetta and B.L. Hu, Phys. Rev. D **52**, 6770 (1995).

[17] E. Calzetta and S. Gonorazky, 'Primordial Fluctuations and Nonlinear Couplings', gr-qc/9608057 (1996).

[18] A. Matacz, 'A New Theory of Stochastic Inflation', gr-qc/9604022 (1996).

[19] A. Matacz, 'Inflation and the Fine-Tuning Problem', (1996).